A REGENERATIVE CARBON DIOXIDE GAS DYNAMIC LASER

Loran Ernest Rhine

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A REGENERATIVE CARBON DIOXIDE
GAS DYNAMIC LASER

by

Loran Ernest Rhine

June 1975

Thesis Advisor: A.E. Fuhs

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Regenerative Carbon Dioxide Gas Dynamic Laser		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis; June 1975
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Loran Ernest Rhine		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE June 1975
		13. NUMBER OF PAGES 44
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Regenerative
Gas Dynamic Laser
Carbon Dioxide Laser

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Performance improvement of a carbon dioxide gas-dynamic laser, due to regeneration, was studied theoretically. The study determined to what extent preheating the nitrogen, before mixing it with the carbon dioxide and water, would effect available laser output energy. Combustion chamber pressure and nitrogen inlet temperature were incremented by 100 psia and 300 K respectively. Due to the fact that little

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(20. ABSTRACT Continued)

dissociation occurred at temperatures studied, no pressure dependence was found for the throat or combustion chamber temperatures. A method of preheating or regenerating the nitrogen is to use a gas-to-gas heat exchanger at the exhaust of the diffuser. Small signal gain was improved from 0.35 to 0.78 m^{-1}. Limiting power extraction was increased by 42%. Optimum nitrogen temperatures were determined to be 1350 K for small signal gain and 2400 K for optimum power extraction. For typical operating conditions the fractional pressure loss across the heat exchanger was found to be 0.1 atmospheres.

A Regenerative Carbon Dioxide Gas Dynamic Laser

by

Loran Ernest Rhine
Lieutenant, United States Navy
B.S., University of New Mexico, 1969

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN PHYSICS

from the

NAVAL POSTGRADUATE SCHOOL
June 1975

ABSTRACT

Performance improvement of a carbon dioxide gas-dynamic laser, due to regeneration, was studied theoretically. The study determined to what extent preheating the nitrogen, before mixing it with the carbon dixoide and water, would effect available laser output energy. Combustion chamber pressure and nitrogen inlet temperature were incremented by 100 psia and 300 K respectively. Due to the fact that little dissociation occurred at temperatures studied, no pressure dependence was found for the throat or combustion chamber temperatures. A method of preheating or regenerating the nitrogen is to use a gas-to-gas heat exchanger at the exhaust of the diffuser. Small signal gain was improved from 0.35 to 0.78 m^{-1}. Limiting power extraction was increased by 42%. Optimum nitrogen temperatures were determined to be 1350 K for small signal gain and 2400 K for optimum power extraction. For typical operating conditions the fractional pressure loss across the heat exchanger was found to be 0.1 atmospheres.

TABLE OF CONTENTS

LIST OF SYMBOLS

A - exchanger total heat transfer area on one side. (ft^2)

A_c - exchanger free flow area. (ft^2)

A_v - Avagadro's Number (6.02×10^{23} $Mole^{-1}$)

C_p - constant pressure specific heat

f - Fanning friction factor

G - exchanger flow stream mass velocity [lbs/(Hr-Ft^2)]

g - gravitational constant [32.2 (ft/sec^2)($\frac{lb}{lb_m}$)]

h - Planck's constant [6.62×10^{-34} J-Sec]

I_o - saturation intensity (Kw/m^2)

I - output beam intensity (Kw/m^2)

J - rotational quantum number.

k - Boltzmann's constant [1.38×10^{-23} J/K]

K_c - contraction loss coefficient for flow at heat exchanger entrance.

K_e - expansion loss coefficient for flow at heat exchanger exit.

M - Mach number

m_{xx} - mass of one molecule of xx.

$\dot{m}_{xx}$ - mass flow rate of xx.

P_4 - pressure in cavity (region 4 in Figure 1) psia

$\dot{q}$ - rate of heat transfer from hot gases to cold nitrogen.

R - universal gas constant

T_N - nitrogen temperature (K)

T_1 - combustion chamber temperature (K)

T_2 - mixing chamber temperature $T_2 = T_1$ (K)

T_3 - nozzle throat temperature (K)

T_4 - translational-rotational temperature in the cavity (K)

v_1 - upstream specific volume of gases with respect to heat exchanger.

v_2 - downstream specific volume of gases with respect to heat exchanger.

v_m - mean specific volume $= \frac{v_1 + v_2}{2}$

α_o - small signal gain coefficient (m^{-1})

γ - $C_p/C_v = 1.4$ for this mixture of gases

Σ_o - effective optical cross section

σ - ratio of free flow area to frontal area

σ_{xx} - optical broadening collisional cross-section of xx (cm^2)

θ_1 - characteristic vibration temperature of CO_2 for ν_1. (1995 K)

θ_2 - characteristic vibration temperature of CO_2 for ν_2. (960 K)

θ_3 - characteristic vibration temperature of N_2 (3380 K)

θ_r - characteristic rotational temperature of CO_2 (2.88 K)

ψ_{xx} - mole fraction of xx.

ρ - density of gases

ν - laser frequency (2.83×10^{13}) Hz

ν_c - optical collision frequency

τ - radiative lifetime (5.38 sec)

μ_{xx} - reduced mass of xx

λ - wavelength of laser radiation (10.6×10^{-6}m)

ACKNOWLEDGMENTS

A hearty Thank You is extended to Distinguished Professor Allen E. Fuhs for his help and guidance in preparing this thesis.

To Sanford Gordon and Bonnie McBride of the NASA Lewis Research Center, thank you very much for your assistance and for the use of your computer program.

To Dr. James Ortwerth of the Air Force Weapons Lab for his suggestion of using a Heat Exchanger to Preheat the nitrogen gas, thank you.

A personal thank you is extended to the staff of the W.R. Church Computer Center for their kind cooperation and help.

To professor J.N. Cooper for his guidance as second reader, thank you.

A special thank you is extended to my wife, Sandy, for her help and moral support.

I. INTRODUCTION

The carbon dioxide gas dynamic laser has been considered a prime candidate for many high energy laser programs. One problem that has always plagued the high energy laser program is that of excessive weight and volume. These presently are the factors that are impeding the development of a tactical Airborne laser weapon system. A heat exchanger can be placed at the exhaust of the diffuser to preheat the nitrogen gas. This system significantly reduces the weight and volume of a given gas dynamic system while maintaining a constant level of energy output.

The combustion chamber properties and nozzle throat conditions were determined by using a computer program developed by Sanford Gordon and Bonnie McBride at the NASA Lewis Research Center in Cleveland, Ohio. The Program and a detailed description of its use are contained in reference [1]. Although the program was written with a rocket engine analysis in mind, it is adequate for computing equilibrium and frozen composition conditions in combustion gases used in gas dynamic lasers.

After the computer determined conditions up to and including the nozzle throat, hand calculations were made using methods specified in reference [2]. According to calculations, the maximum available energy can be increased

by a factor of 5 to 10. The small signal gain as a function of nitrogen temperature reached a maximum value of 0.773. This maximum indicates a best operating temperature.

The diffuser is used to recover the pressure in the flow stream and must be matched to the heat exchanger.

A gas to gas heat exchanger is used to insure that the proper temperature difference is attained from the cold nitrogen side to the hot nitrogen side (see Figure 1). The pressure must not be allowed to decrease to atmospheric pressure at mean sea level. This can be done by use of procedures outlined in references [3], [4], and [5].

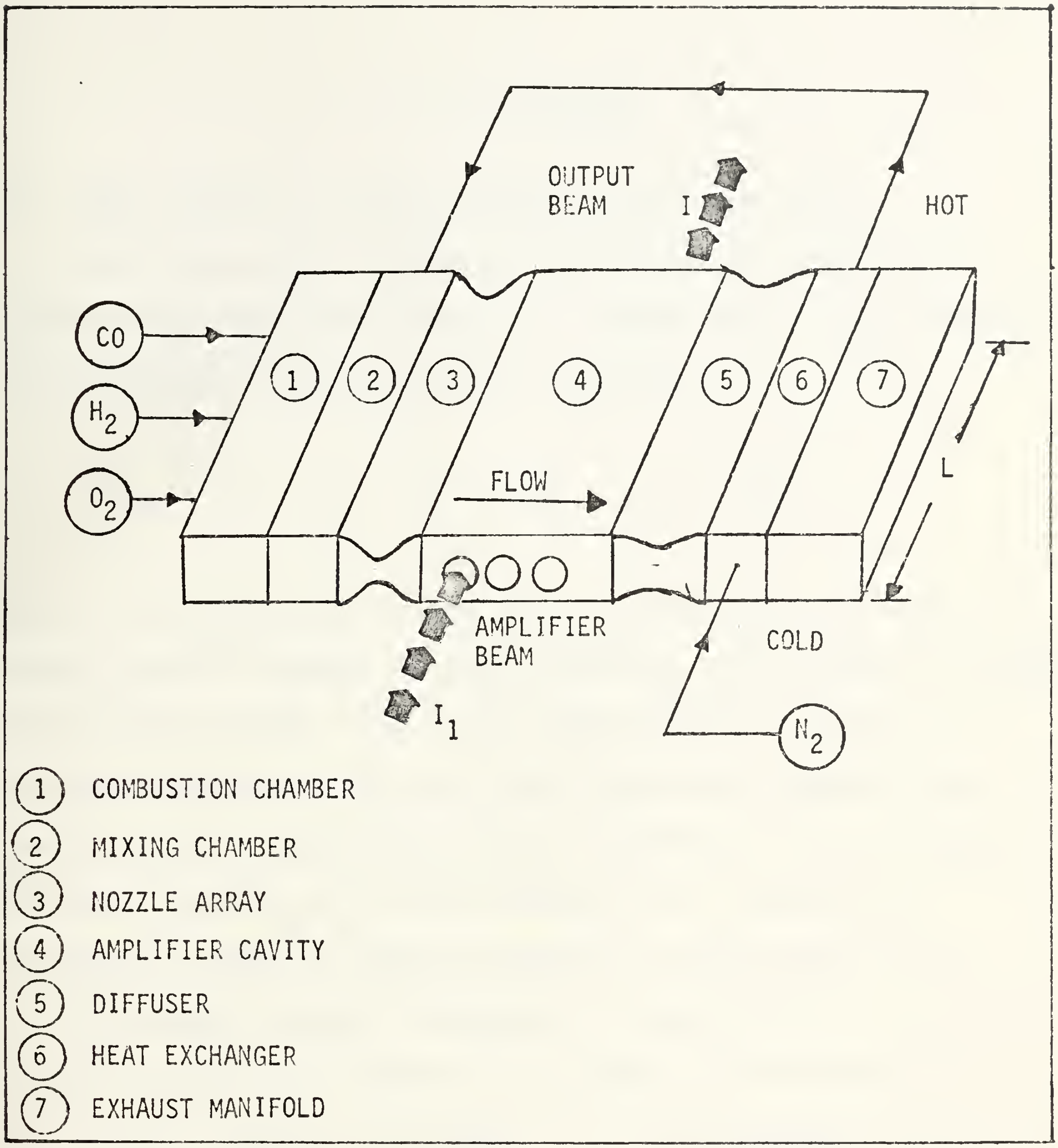


Figure 1 BASIC ARRANGEMENT FOR A REGENERATIVE GAS DYNAMIC LASER WITH MOPA CONFIGURATION.

II. THEORY

A. BASIC THEORY OF A GAS DYNAMIC LASER AMPLIFIER

Carbon monoxide, hydrogen and oxygen are fed into the combustion chamber and ignited. The amount of each reactant is determined by the following reaction;

$$90N_2 + 9CO + H_2 + 5O_2 \rightarrow 9CO_2 + H_2O + 90N_2$$

After the reactant products reach equilibrium, nitrogen is mixed with the carbon dioxide and water in the mixing chamber. This yields a molar mixture of 90% nitrogen, 1% water and 9% carbon dioxide. This is the composition required for operation of a gas dynamic laser amplifier. It is assumed that the ratio C_p/C_v of the mixture is 1.4 while the vibrational mode of carbon dioxide is frozen out at T_3.

The nozzle array is designed to attain Mach 1.0 at the throat (region 3 in Figure 1) and Mach 4.0 or greater inside the cavity (region 4 in Figure 1). For purposes of this thesis, all calculations have been done assuming that a Mach number of 4.0 exists inside the amplifier cavity. The technique used to determine the area ratios is described in detail in reference [6]. Since these calculations are well documented, only some of the final results will be mentioned.

From the equilibrium combustion temperature, the throat and cavity temperatures can be determined and the small signal gain can be calculated.

The small signal gain is applicable only inside the cavity (region 4 in Figure 1). Assuming that the oscillator and cavity are matched at a wavelength of 10.6 micrometers, the output beam intensity is given by Equation 1.

$$I = I_1 e^{\alpha \ell} \tag{1}$$

α is the gain coefficient and ℓ is the overall beam pathlength inside the cavity. ℓ is found from Equation 2.

$$\ell = NL \tag{2}$$

N is the number of times that the beam traverses the width L of the cavity (see Figure 1).

A measure of the power that can be extracted, defined in Equation 5.23 of reference [2] as the limiting power extraction, is $\alpha_o I_o$ where I_o is defined by Equation 3.

$$I_o = \frac{k(\theta_3 - \theta_1) Q_{VIB}}{(\frac{\varepsilon_1}{\varepsilon_3} \tau_1 + \tau_4) \varepsilon_3 \Sigma_o} \tag{3}$$

The maximum available energy in the excited state of the gases is described in reference [2] and can be calculated

with Equation 4. Table II shows the result of calculations at incremental temperatures.

$$E_{max} = \frac{1}{\bar{m}} \left[\frac{h\nu}{e^{\frac{3380}{T_3}} - 1} \right] \quad (4)$$

$\bar{m}$ is defined as the average mass of the gas molecules. $\bar{m}$ is calculated with Equation 5.

$$\bar{m} = \frac{\psi_{CO_2} M_{CO_2} + \psi_{H_2O} M_{H_2O} + \psi_{N_2} M_{N_2}}{A_V} \quad (5)$$

ψ is the mole fraction of the subscripted quantity.

As a basis for determining the change in limiting power output, small signal gain and maximum available energy, an assumption was made that all gases were at a room temperature of 300 K, prior to combustion. The nitrogen temperature was then incremented and new calculations made at each value of nitrogen temperature. These new calculations are compared to those at 300 K in tabular and graphical form.

B. SMALL SIGNAL GAIN

Reference [2] outlines procedures for determining the small signal gain coefficient. The overall gain I/I_1 may be determined from Equation 1 after α_o has been found from Equation 6. Pages 5-12 of reference [2] shows the relation between α and α_o.

$$\alpha_o = \frac{1}{4\pi} \frac{\lambda_o^2}{\tau \nu_c} \frac{(2J+1)}{T_4/2\theta_r} e^{\frac{-J(J+1)\theta_r}{T_4}} \left[\frac{n(001)-n(100)}{\psi_{CO_2}N}\right] \psi_{CO_2}N \qquad (6)$$

The population inversion is determined by the quantity in brackets in Equation 6. Population inversion is related to cavity and throat temperature by Equation 7. It is assumed that the vibrational temperatures of nitrogen and carbon dioxide are frozen at T_3. Appendix B is a derivation for ν_c in terms of temperature and number density.

$$\left[\frac{n(001)-n(100)}{\psi_{CO_2}N}\right] = \left[\frac{e^{-\theta_3/T_3} - e^{-\theta_1/T_4}}{Q_{VIB}}\right] \qquad (7)$$

Here, the temperature T_3 is frozen at the nozzle throat and the system is operating so that the threshold condition is exceeded. That is, $\theta_3/\theta_1 < T_3/T_4$. This condition insures that the small signal gain will be positive. Q_{vib} is related to throat and cavity temperatures by Equation 8.

$$Q_{vib} = (1-e^{-\theta_1/T_4})^{-1}(1-e^{-\theta_2/T_4})^{-2}(1-e^{-\theta_3/T_3})^{-1} \qquad (8)$$

C. MAXIMUM AVAILABLE ENERGY

The maximum available energy is dependent upon the wavelength of the laser and the vibrational temperature of the transition level for stimulated emission. E_{max} can be

calculated using Equation 4 or Equation 9. Equation 9 is the same as Equation 4 except that $\frac{c}{\lambda}$ has been substituted for ν.

$$E_{max} = \frac{1}{\bar{m}} \frac{hc}{\lambda} (e^{\theta_3/T_3} - 1)^{-1} \quad (9)$$

D. HEAT EXCHANGER

The heat exchanger is the key to success of this system. It takes energy from the exhaust gases at the diffuser and transfers it to the nitrogen gas. This process allows the nitrogen to be heated to an optimum temperature. Equation 13 describes the pressure drop across the exchanger. Since this system is to operate at mean sea level and higher, the minimum pressure on the downstream side (region 7 of Figure 1) of the exchanger must be greater than 1 atmosphere.

Assuming that the nitrogen enters the exchanger at 300 K and exits the exchanger at a temperature of T_N (optimal), the total heat transfered is defined in reference [3] as Equation 10.

$$\dot{q}_N = \dot{m}_N C_{pN} [T_N(HOT - T_N(COLD)] \quad (10)$$

$\dot{q}_N$ must equal the heat given up by the mixture flowing through the heat exchanger. This is described by Equation 11.

$$\dot{q}_N = \dot{q}_m = \dot{m}_m C_{pm} [T_m(HOT) - T_m(COLD)] \quad (11)$$

Therefore, Equations 10 and 11 can be combined to produce Equation 12 which describes the temperature drop across the heat exchanger.

$$T_5 - T_7 = \left(\frac{\dot{m}_N}{\dot{m}_m}\right) \left(\frac{C_{pN}}{C_{pm}}\right) [T_N(HOT - T_N(COLD)] \quad (12)$$

Here, T_5 is defined as the upstream temperature, T_m (hot), and T_7 is the downstream temperature, T_m (cold).

A more detailed analysis is in Appendix A. Equation 13 is used in reference [4] to describe Δp.

$$\Delta p = \frac{G^2 v_1}{2g} \left[(K_c + 1 - \sigma^2) + 2\left(\frac{v_2}{v_1} - 1\right) + f\frac{A}{A_c}\frac{v_m}{v_1} - (1 - \sigma^2 - K_e)\frac{v_2}{v_1}\right] \quad (13)$$

III. CALCULATIONS

A. THE GAS DYNAMIC LASER CAVITY

Calculations for the combustion chamber and nozzle throat were computed by using the NASA Lewis Computer Program, reference [1]. The remaining calculations were done by hand using the isentropic tables in reference [6].

For conditions in region 4, shown in Figure 1, Appendix A of reference [6] was used. At M = 4.0 the following relations are found:

$$\frac{P_4}{P_1} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}} = 0.6586 \times 10^{-2}$$

$$\frac{T_4}{T_1} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1} = 0.2381$$

$$\frac{A}{A^*} = \frac{A_4}{A_{throat}} = 10.72$$

Conditions in region 4 (Figure 1) can be determined from the following relationships:

$$P_4 = \left(\frac{P_4}{P_1}\right) P_1 = (0.6586 \times 10^{-2})\ P_1 \qquad (14)$$

$$T_4 = \left(\frac{T_4}{T_1}\right) T_1 = (0.2381)\ T_1 \qquad (15)$$

$$A_4 = \left(\frac{A_4}{A_{throat}}\right) A_{throat} = (10.72)\ A_{throat} \qquad (16)$$

One model of the starting process uses a normal shock which develops inside the cavity (region 4 of Figure 1). Therefore, the diffuser must be able to swallow this shock before the system will be stable enough to operate effectively. From Appendix B of reference [6] at $M_1 = 4.0$, the Mach number of the flow behind the normal shock is 0.4350. Before the diffuser can swallow the shock, the flow must be accelerated to Mach 1 at the diffuser throat. Therefore from Appendix A of reference [6] at M = .4350, the following ratio is found:

$$\frac{A}{A_N^*} = \frac{A_4}{A_{diffuser}} = 1.5 \qquad (17)$$

Diffuser conditions change after the shock has been swallowed. Therefore, the only noteworthy condition is the fact that the ratio of diffuser throat area to nozzle throat area can now be determined.

$$\frac{A_{diffuser}}{A_{throat}} = \frac{A/A_N^*}{A/A_D^*} = \frac{10.72}{1.5} = 7.15 \qquad (18)$$

By the methods described in Section 3.4 of reference [6], the Mach number at the diffuser throat is now 3.55 and the

normal shock is positioned just past the diffuser throat near region 6 of Figure 1.

B. SMALL SIGNAL GAIN COEFFICIENT

The small signal gain coefficient must be determined before the output intensity can be calculated. Combining Equations 6, 7, and 8 yields Equation 19. Equation 19 is explicit in terms of known throat and cavity temperatures. The constant 1.6×10^5 is the result of inserting $\lambda_o = 10.6 \times 10^{-6}$m, $J = 20$, $\theta_r = 2.88$ K, $\nu_c = 39.5 \times 10^{-14} N \sqrt{T_4}$ and $\tau = 5.38$ sec.

$$\alpha_o = [1.6\times10^5]\left[\frac{e^{\frac{-239.4}{T_4}}}{(T_4)^{3/2}}\right]\left[\frac{e^{\frac{-3380}{T_3}} - e^{\frac{-1995}{T_4}}}{(1-e^{\frac{-1995}{T_4}})^{-1}(1-e^{\frac{-960}{T_4}})^{-2}(1-e^{\frac{-3380}{T_3}})^{-1}}\right] \quad (19)$$

Table 1 gives the values of nitrogen, throat and cavity temperatures used in calculating α_o at the specified nitrogen temperatures. Figure 2 is a graph of α_o versus temperature of the nitrogen. It is shown that an operating nitrogen temperature of approximately 1300 K will maximize the small signal gain at 0.773. See Table II for values of α_o at specified nitrogen temperatures.

C. MAXIMUM AVAILABLE ENERGY

The maximum available energy can be easily calculated by using Equation 4 after $\overline{m}$ has been determined by using Equation 5.

$$\overline{m} = \frac{(.09)(44) + (.01)(18) + (.90)(28)}{6.02 \times 10^{23}} = 4.87 \times 10^{-23} \frac{\text{gm}}{\text{molecule}}$$

$$E_{max} = (3.83 \times 10^{5})\left(e^{\frac{3380}{T_3}} - 1\right)^{-1} \frac{\text{Joules}}{\text{Kgm}}$$

These results are listed in Table II and Figure (3) is a plot of E_{max} versus nitrogen temperature.

D. LIMITED POWER EXTRACTION

The saturation beam intensity, as defined by Equation 3, is a valuable aid in approximating the limiting power extraction. Equation 3 must be used in conjunction with the following equations:

$$\varepsilon_1 = e^{-\theta_1/T_4}\left[\left(e^{\theta_1/T_4}-1\right)^{-1} + 2\frac{\theta_2}{\theta_1}\left(e^{\theta_2/T_4} - 1\right)^{-1}\right]^{-1} \quad (20)$$

$$\varepsilon_3 = \left(1 - e^{-\theta_3/T_3}\right) \quad (21)$$

$$\Sigma_o = \frac{1}{4\pi}\frac{\lambda_o^2}{\tau\nu_c}\frac{(2J+1)}{T_4/2\theta_r} e^{\frac{-\theta_r J(J+1)}{T_4}} \quad (22)$$

Combining Equations 3, 8, 20, 21 and 22 yields Equation 23.

$$I_o = \frac{k(\theta_3-\theta_1)}{(\frac{\varepsilon_1}{\varepsilon_3}\tau_1+\tau_4)} \frac{(1-e^{\frac{-\theta_1}{T_4}})^{-1}(1-e^{\frac{-\theta_2}{T_4}})^{-2}(1-e^{\frac{-\theta_3}{T_3}})^{-1}}{\frac{1}{4\pi}\frac{\lambda_o^2}{\tau\nu_c}\frac{2J+1}{T_4/2\theta_r}e^{-\theta_r J(J+1)/T_4}} \tag{23}$$

The product of Equation 19 and Equation 23 is Equation 24.

$$\frac{\alpha_o I_o}{P_4^2} = [10\psi_{CO_2}][\frac{\theta_3-\theta_1}{T_4}][\frac{e^{\frac{-\theta_3}{T_3}} - e^{\frac{-\theta_1}{T_4}}}{\varepsilon_3(\frac{\varepsilon_1}{\varepsilon_3}\tau_1P_4+\tau_4P_4)}] \tag{24}$$

$\alpha_o I_o/P_4^2$ is in Mks units. τ_1P_4 and τ_4P_4 can be found in Figures 2.8 and 2.10 of reference [2]. Table III shows the calculated results from Equation 24 and Figure 4 is a plot of $\alpha_o I_o/P_4^2$ versus nitrogen temperature.

E. HEAT EXCHANGER

Using the optimum temperature as shown in Figure 4 the hot nitrogen temperature is specified as 2400 K. This gives a temperature difference in the nitrogen of 2100 K. An energy balance will produce an equation relating the temperature difference from the hot to the cold side of the heat exchanger. From equation 10 and C_{pN} equal to 0.39, the rate of heat transfer is

$$\dot{q} = \dot{m}_N C_{pN} (\Delta T_N) = \dot{m}_N (819) \frac{BTU}{lbm\text{-}sec}.$$

Reference [7] contains values for C_p. From Equation 12, the temperature difference is

$$T_5 - T_7 = (1803) \frac{C_{pN}}{C_{pm}} \approx 1617 \text{ K}$$

A more detailed analysis of the heat exchanger is in Appendix [A].

F. APPROXIMATE WEIGHT AND VOLUME

Assuming one-second bursts of energy in the output, the amount of fuel needed for any pre-determined number of bursts can be calculated easiest at the nozzle throat where gas properties are well defined. From reference [6], the mass flow rate is

$$\dot{m} = UA\rho$$

where U is the mass velocity, A is the area of the nozzle and ρ is the density of the gases.

The limiting power per unit mass is

$$\frac{\alpha_o I_o}{\tilde{m}} = \overline{W}$$

where $\tilde{m} = (1 \text{ sec})\ \dot{m}_m$. If $\overline{W}$ increases by some factor A and $\tilde{m}$ remains constant then $\alpha_o I_o$ must have increased by A also. It is desired to maintain $\overline{W}$ constant and adjust the mass necessary to compensate for this increase in output.

$$\overline{W}(1 + A) = \frac{(1 + A)\ \alpha_o I_o}{\tilde{m}}$$

$$= \frac{\alpha_o I_o}{\frac{\tilde{m}}{(1+A)}} = \frac{\alpha_o I_o}{\tilde{m}'} \rightarrow \tilde{m}' = \frac{\tilde{m}}{(1+A)}$$

This means that the mass necessary to maintain a constant output can be decreased by a factor of $\frac{A}{(1+A)}$. Calculations have shown that A = 0.42 and the fuel necessary is only 70% of the fuel used without a regenerator. Therefore a savings in fuel weight and volume of 30% has been found.

IV. RECOMMENDATIONS AND CONCLUSIONS

It has been shown that pre-heating the nitrogen gas prior to mixing it with the combusted products of hydrogen, oxygen, and carbon monoxide increases the gain of a carbon dioxide gas dynamic laser from 0.35 m^{-1} to 0.77 m^{-1}. The limiting power extracted reaches a maximum at a temperature of 2400 K. This produces an optimum operating temperature and the limiting power extracted can be increased by a factor of 1.42. This implies an increase of 42% in output power while maintaining a constant mass flow rate. This means that a constant level of output power can be maintained while reducing the amount of fuels needed by 30%. This decreases the amount of storage volume by 30%.

It is recommended that a further study be conducted to determine the effects that a heat exchanger would have on other fuels to produce a 10.6 micrometer wavelength with a possible extension into other wavelengths.

TABLE I

TEMPERATURES AND PRESSURES OF SYSTEM

T_N (K)	T_2 (K)	T_3 (K)	T_4 (K)	$P_1=P_2$(PSIA)	P_4(PSIA)
300	1076	928	256	500	3.293
				600	3.951
				700	4.610
				800	5.268
				900	5.927
				1000	6.586
600	1303	1130	310	500	3.293
				600	3.951
				700	4.610
				800	5.268
				900	5.927
				1000	6.586
900	1535	1338	365	500	3.293
				600	3.951
				700	4.610
				800	5.268
				900	5.927
				1000	6.586
1200	1774	1553	422	500	3.293
				600	3.951
				700	4.610
				800	5.268
				900	5.927
				1000	6.586
1250	1815	1590	432	500	3.293
				600	3.591
				700	4.610
				800	5.268
				900	5.927
				1000	6.586
1300	1855	1623	442	500	3.293
				600	3.951
				700	4.610
				800	5.268
				900	5.927
				1000	6.586
1350	1895	1662	451	500	3.293
				600	3.951
				700	4.610
				800	5.268
				900	5.927
				1000	6.586
1400	1935	1694	461	500	3.293
				600	3.951
				700	4.610
				800	5.268
				900	5.927
				1000	6.586

TABLE I (Continued)

T_N (K)	T_2 (K)	T_3 (K)	T_4 (K)	$P_1=P_2$(PSIA)	P_4(PSIA)
1450	1975	1730	470	500	3.293
				600	3.951
				700	4.610
				800	5.268
				900	5.927
				1000	6.586
1500	2014	1772	480	500	3.293
				600	3.951
				700	4.610
				800	5.268
				900	5.927
				1000	6.586
1800	2246	1991	535	500	3.293
				600	3.951
				700	4.610
				800	5.268
				900	5.927
				1000	6.586
2100	2463	2204	586	500	3.293
				600	3.951
				700	4.610
				800	5.268
				900	5.927
				1000	6.586
2400	2662	2404	634	500	3.293
				600	3.951
				700	4.610
				800	5.268
				900	5.927
				1000	6.586
2700	2845	2590	677	500	3.293
				600	3.951
				700	4.610
				800	5.268
				900	5.927
				1000	6.586
3000	3017	2760	718	500	3.293
				600	3.951
				700	4.610
				800	5.268
				900	5.927
				1000	6.586

TABLE II

COMPUTED VALUES FOR E_{max} & α_o

T_N (K)	(KJ/KG) ← E_{max}	→ (KJ/lbm)	$\alpha_o(m^{-1})$
300	10.3	4.67	0.36
600	20.25	9.12	0.57
900	33.3	15.1	0.71
1200	49	22.2	0.772
1250	51.9	23.6	0.773
1300	54.5	24.7	0.772
1350	57.7	26.2	0.771
1400	60.3	27.4	0.771
1450	63.3	28.7	0.769
1500	66.8	30.3	0.768
1800	85.9	38.9	0.728
2100	105	47.6	0.67
2400	124	56.3	0.61
2700	143	64.9	0.546
3000	159	72.1	0.495

TABLE III

COMPUTED VALUES OF $\alpha_o I_o / P_4^2$ $(\frac{MW}{m^3})(\frac{1}{N/m^2})^2$

T_N (K)	T_3 (K)	T_4 (K)	$\alpha_o I_o / P_4^2$
300	928	256	0.543
600	1130	310	0.65
900	1338	365	0.746
1200	1553	422	0.827
1350	1662	451	0.844
1500	1772	480	0.8834
1800	1991	535	0.918
2100	2204	586	0.952
2400	2404	634	0.963
2700	2590	677	0.964
3000	2760	718	0.960

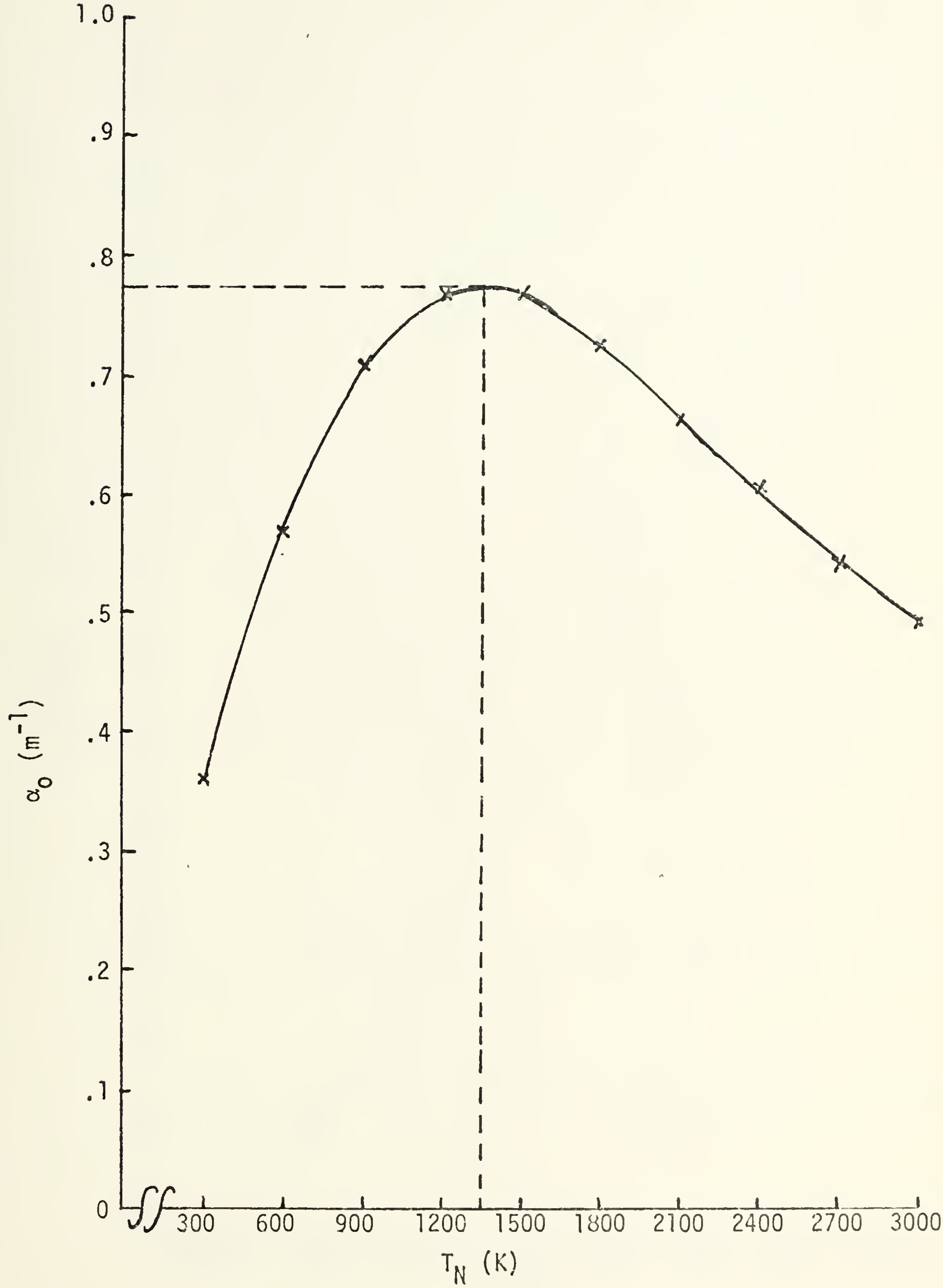


Figure 2 SMALL SIGNAL GAIN

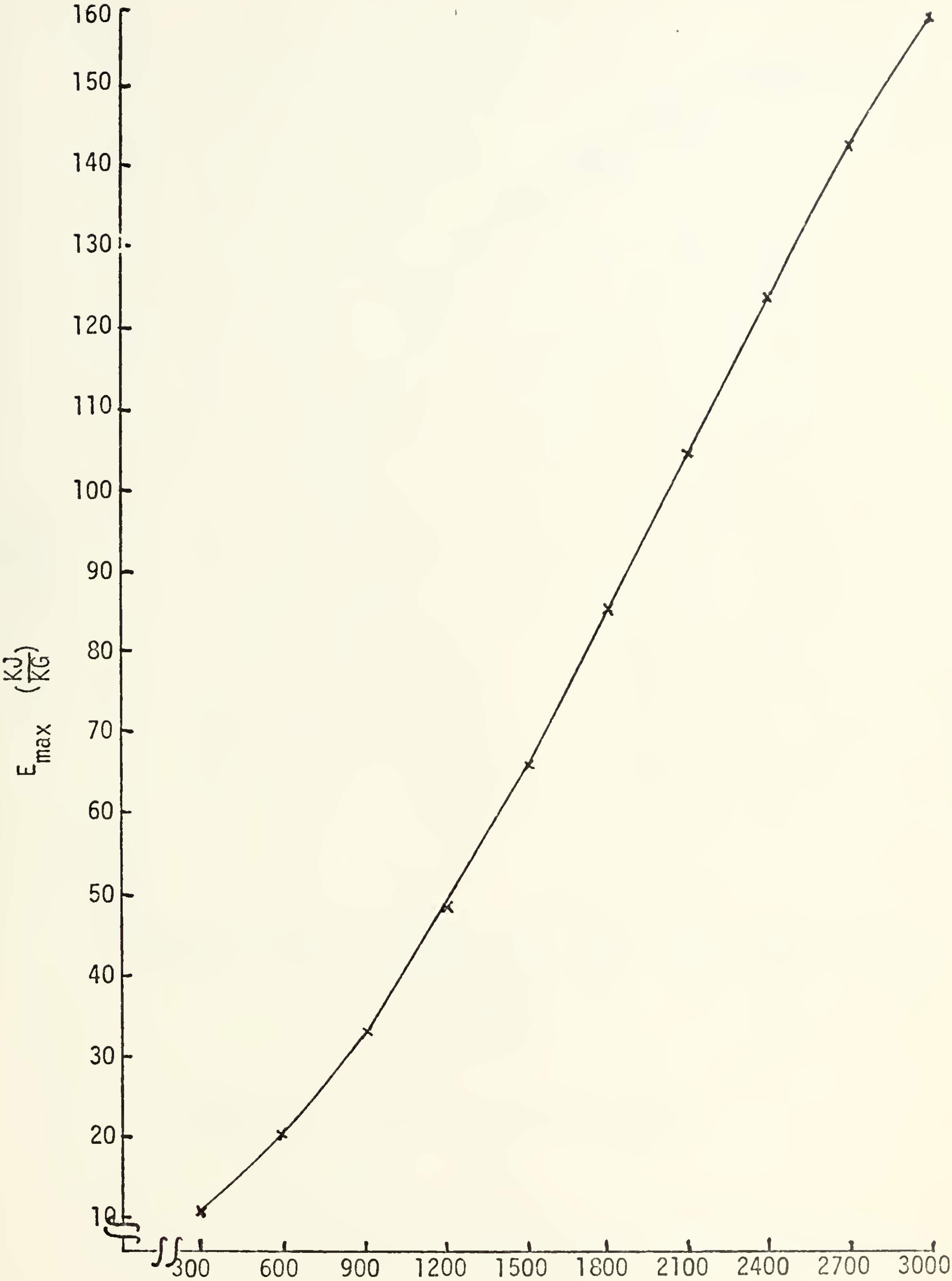


Figure 3 MAXIMUM AVAILABLE ENERGY

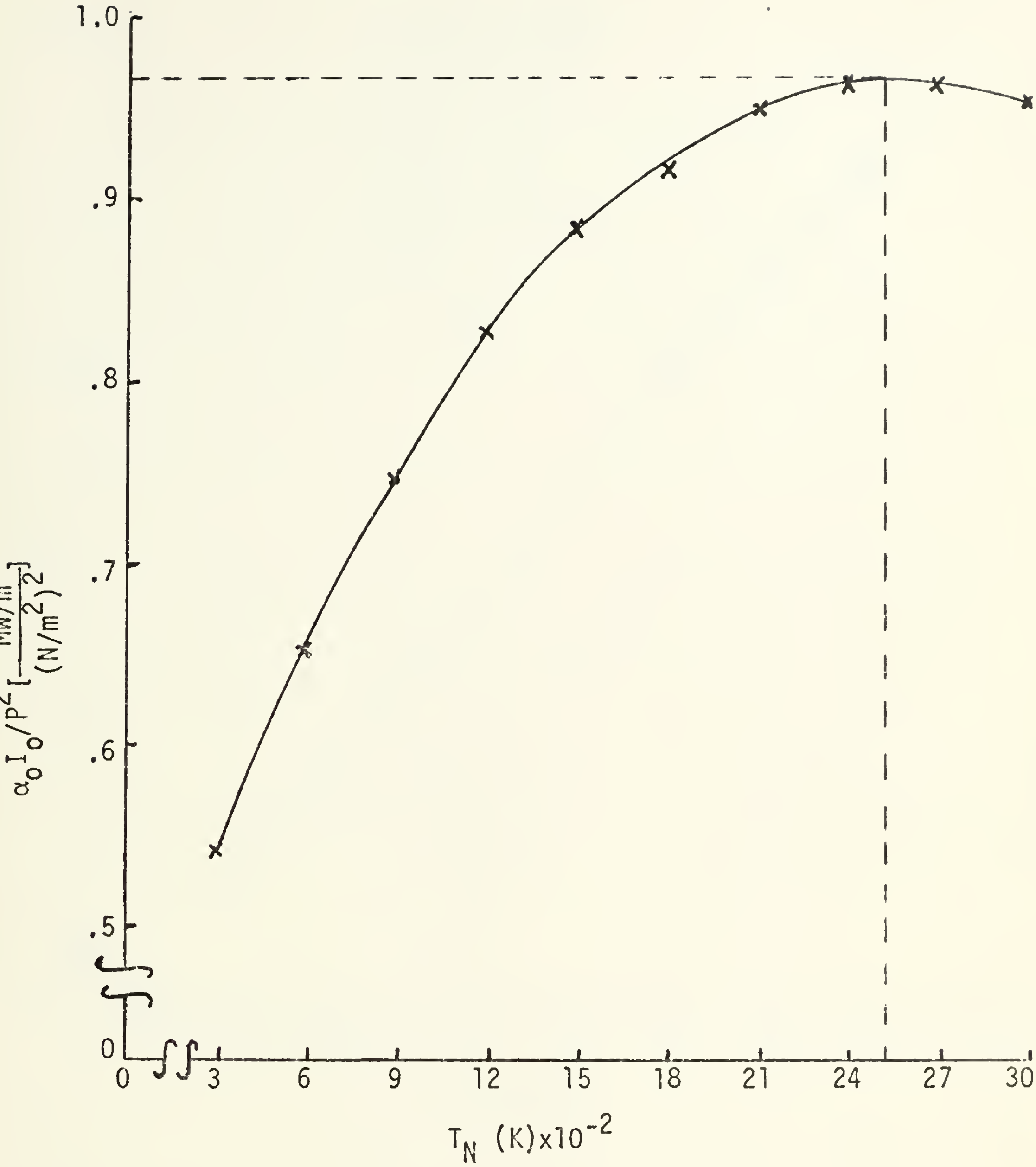


Figure 4 LIMITING POWER OUTPUT

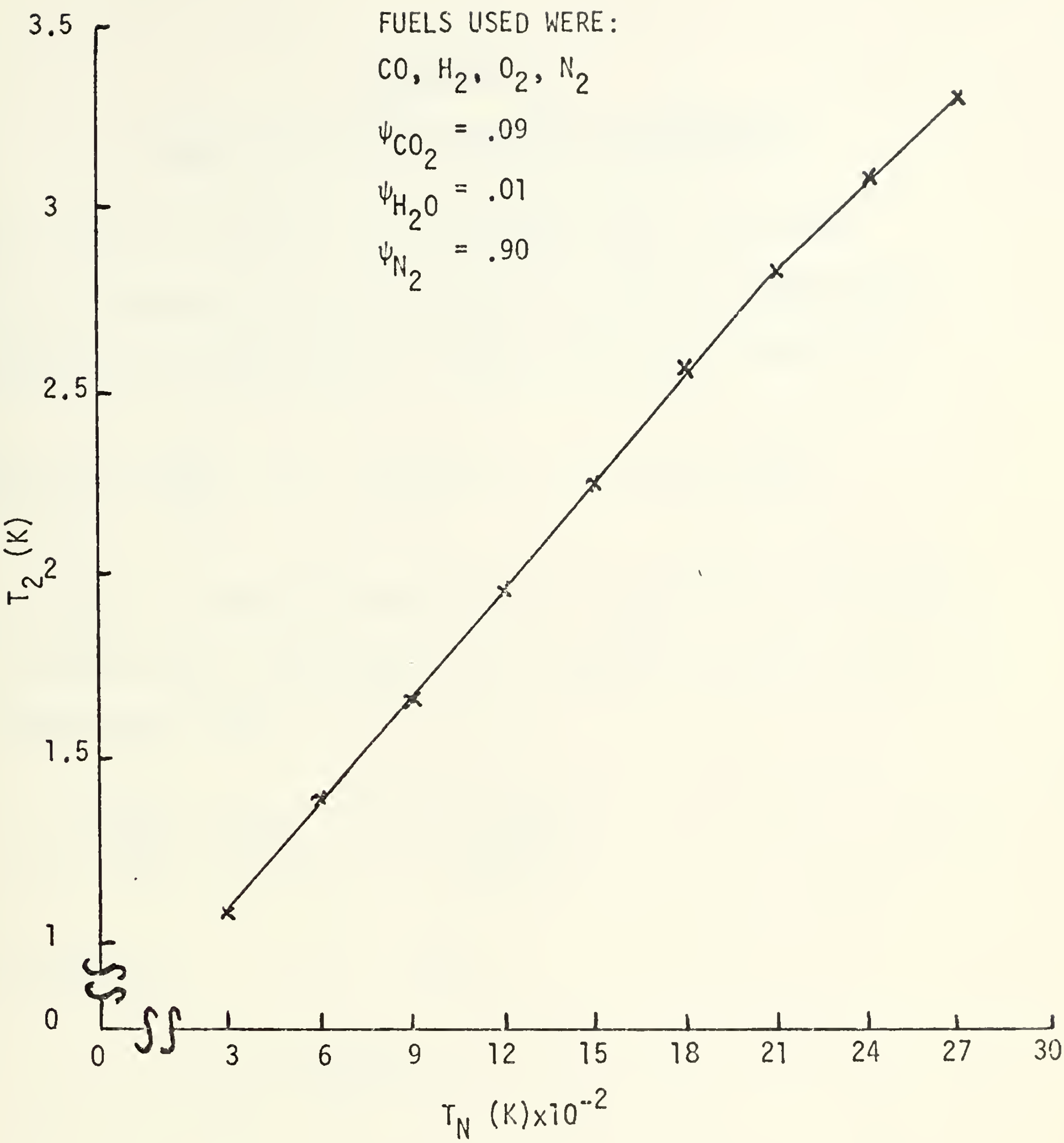


Figure 5 MIXING CHAMBER EQUILIBRIUM TEMPERATURE

APPENDIX A

THEORY OF GAS-TO-GAS HEAT EXCHANGER

As stated on page 21 of reference [4], the terms involving exchanger input and output flow are negligible. An inspection of Equation 13, shows that the assumption

$$f \frac{A}{A_c} \frac{v_m}{v_1} + 2\left(\frac{v_2}{v_1} - 1\right) >> (K_c + 1 - \sigma^2) - (1 - \sigma^2 - K_e) \frac{v_2}{v_1}$$

can be used to develop Equation 4'. $K_c + 1 - \sigma^2$ is associated with the upstream flow while $(1 - \sigma^2 - K_e) \frac{v_2}{v_1}$ is associated with the downstream flow.

$$\Delta P = \frac{G^2 v_1}{2g} \left[2\left(\frac{v_2}{v_1} - 1\right) + f \frac{A}{A_c} \frac{v_m}{v_1}\right] \qquad (4')$$

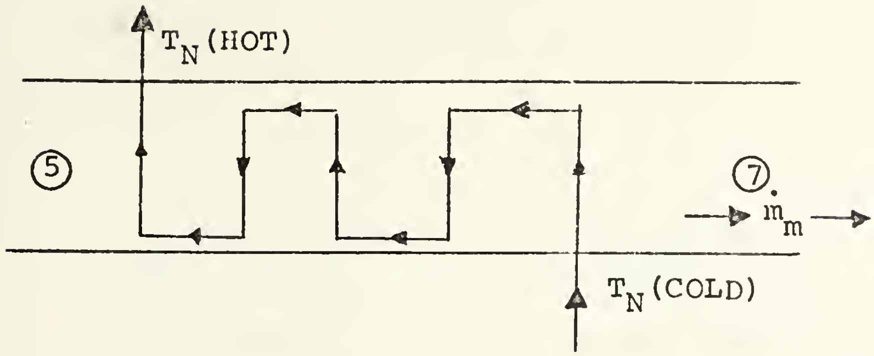
By definition:

$$v_m = \frac{v_1 + v_2}{2} .$$

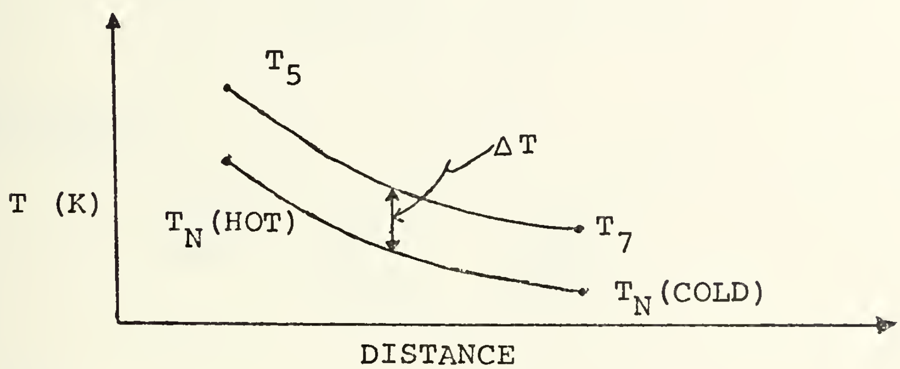
Therefore:

$$\Delta P = \frac{G^2 v_1}{2g} \left[\frac{v_2}{v_1}\left(\frac{f}{2} \frac{A}{A_c} + 2\right) + \left(\frac{f}{2} \frac{A}{A_c} - 2\right)\right]$$

Now, the heat transfer area is needed. The derivation is based on the counterflow heat exchanger, from reference [7], shown below.



The temperature variation as a function of distance through the heat exchanger is shown below.



Now:

$$\dot{q} = hA\Delta T_m \quad \left(\frac{BTU}{sec}\right)$$

ΔT_m = logarithmic mean temperature difference.

h = heat transfer coefficient.

A = transfer area

Before A can be determined, ΔT_m is needed. Therefore:

$$d(\Delta T) = dT_m + dT_N$$

where:

dT_m = change in hot gas temperature

dT_N = change in cold gas temperature

Now:

$$d\dot{q} = -C_m \, \dot{m}_m \, dT_m = C_N \, \dot{m}_N \, dT_N$$

Therefore:

$$d(\Delta T) = \frac{d\dot{q}}{-C_m \, \dot{m}_m} + \frac{d\dot{q}}{C_N \, \dot{m}_N}$$

Also:

$$d\dot{q} = h \Delta T \, dA$$

Therefore:

$$\frac{d(\Delta T)}{(\Delta T)} = h\left(\frac{1}{C_N \, \dot{m}_N} - \frac{1}{C_m \, \dot{m}_m}\right) dA$$

Integration of this equation leads to

$$\ln\left[\frac{T_7 - T_N \,(\text{COLD})}{T_5 - T_N \,(\text{HOT})}\right] = hA\left(\frac{1}{C_N \, \dot{m}_N} - \frac{1}{C_m \, \dot{m}_m}\right)$$

As shown previously

$$hA = \dot{q}/\Delta T_m$$

and

$$\dot{q}/\nabla T_m = \frac{\dot{m}_m C_m (T_5 - T_7)}{\Delta T_m} = \frac{\dot{m}_N C_N [T_N(\text{HOT}) - T_N(\text{COLD})]}{\Delta T_m}$$

Now:

$$\ln\left[\frac{T_7 - T_N(\text{COLD})}{T_5 - T_N(\text{HOT})}\right] = \frac{T_N(\text{HOT}) - T_N(\text{COLD}) - T_5 + T_7}{\Delta T_m}$$

and

$$\Delta T_m = \frac{[T_N(\text{HOT}) - T_N(\text{COLD}) - T_5 + T_7]}{\ln\left[\frac{T_7 - T_N(\text{COLD})}{T_5 - T_N(\text{HOT})}\right]}$$

Now:

$$A = \frac{\dot{m}_m C_m (T_5 - T_7)}{h \Delta T_m} = \frac{\rho_5 \; U_5 \; A_c C_m (T_5 - T_7)}{h \Delta T_m}$$

And the ratio A/A_c can be found as

$$[\rho_5 U_5 C_m (T_5 - T_7)]/[h \Delta T_m]$$

Assuming a 10% pressure drop across the exchanger, P_5 equal to 1.1 atmospheres and T_5 = 4500 °R, the Reynolds number can be calculated.

$$R_e = \frac{D_o U_5 \rho_5}{\mu}$$

Assuming a tube diameter of .1 ft, U_5 = 988 (ft/sec), $\rho = 3.02 \times 10^{-4} \frac{\text{SLUG}}{\text{ft}^3}$ and $\mu = 1.09 \times 10^{-7} \frac{\text{SLUG}}{\text{ft-sec}}$;

$$R_e = 2.73 \times 10^5$$

From Figure 6-9 of reference [3]

$$\frac{hD_o}{k} = 5.75 \times 10^2$$

$$k = k_{ref} \sqrt{T/T_{ref}}$$

where k_{ref} and T_{ref} are from Table a-5 of reference [3].

$$k = (0.0337) \sqrt{\frac{4500}{1500}} = .05 \frac{\text{BTU}}{\text{hr ft °R}}$$

$$h = 336 \text{ BTU/(hr ft}^2 \text{ °R)}$$

$$\frac{A}{A_c} = \frac{\rho_5 U_5 C_m}{h \Delta T_m} (T_5 - T_7) \approx 12$$

With these quantities and $\frac{v_2}{v_1}$ equal to 1.36 and f = 0.02 (from Figure 6-3 of reference [4] and $G^2 v_1 = 1.365 \times 10^4$ $lb^2/(lbm\ ft\ sec^2)$;

$$\Delta P \approx \frac{G^2 v_1}{2g} = 0.1 \text{ atmospheres} = 212 \frac{lb}{ft^2}$$

APPENDIX B

DERIVATION OF ν_c

From Equation 3.7 of reference [2], ν_c is

$$\sum_i N_i \sigma_i \bar{C}_i = N[\psi_{CO_2}\sigma_{CO_2}\bar{C}_{CO_2} + \psi_{N_2}\sigma_{N_2}\bar{C}_{N_2} + \psi_{H_2O}\sigma_{H_2O}\bar{C}_{H_2O}]$$

where

ψ = the mole fraction

σ is the optical broadening collisional cross section.

$\bar{C}_i$ is the relative mean speed.

$\psi_{CO_2} = .09 \qquad \psi_{N_2} = .90 \qquad \psi_{H_2O} = .01$

From pages 3-5 of reference [2]

$\sigma_{CO_2} = 1.3 \times 10^{-14}\ cm^2$

$\sigma_{N_2} = .87 \times 10^{-14}\ cm^2$

$\sigma_{H_2O} = .38 \times 10^{-14}\ cm^2$

From page 3.4 of reference [2]

$$\bar{C}_i = \sqrt{\frac{8\ R\ T_4}{\pi \mu}}$$

where

$$\mu = M_{CO_2} \quad \text{for } CO_2 \text{ collisions}$$

$$\mu = \frac{M_{CO_2}\ M_x}{M_{CO_2} + M_x} \quad \text{for } H_2O \text{ and } N_2 \text{ collisions.}$$

Now:

$$\bar{C}_{CO_2} = \sqrt{\frac{8\ K\ T_4}{\pi\ M_{CO_2}}} = 21\ \sqrt{T_4}\ \ \frac{M}{sec}$$

$$\bar{C}_{N_2} = \sqrt{\frac{8\ R\ T_4}{\pi\ \mu_{N_2}}} = 44.2\sqrt{T_4}\ \ \frac{M}{sec}$$

$$\bar{C}_{H_2O} = \sqrt{\frac{8\ R\ T_4}{\pi \cdot \mu_{H_2O}}} = 40\ \sqrt{T_4}\ \ \frac{M}{sec}$$

Therefore:

$$\nu_c = N\ \sqrt{T_4}\ \times 10^{-14}[(21)(.09)(1.3) + (44.2)(.9)(.87) + (40)(.01)(.38)]$$

$$= 3.95 \times 10^{-13}\ N\ \sqrt{T_4}$$

LIST OF REFERENCES

1. NASA Lewis Research Center, NASA SP-273, Computer Program for Calculations of Complex Chemical Equilibrium Compositions, Rocket Performance Incident and Reflected Shocks, and Chapman-Jouguet Detonations, by Sanford Gordan and Bonnie McBride, 1971.

2. Hertzberg, A., Christiansen, W., and Russell, D., AIAA Professional Study Series, Carbon Dioxide Gas Dynamic Laser, University of Washington, Seattle, July 1973.

3. Holman, J.P., Heat Transfer, P.225-249. McGraw-Hill, 1963.

4. Kays, W.M., Gas Turbine Plant Exchangers, Chapter 5 p. 20 and p. 68, American Society of Mechanical Engineers, 1951.

5. Kays, W.M., and London, A.L., Compact Heat Exchangers, Chapter 5, McGraw-Hill, 1964.

6. John, James E.A., Gas Dynamics, p. 353-363, Allyn Bacon Inc., 1969.

7. Faines, V.M., Applied Thermodynamics, Macmillan Company, New York, 1949.

BIBLIOGRAPHY

1. Air Force Weapons Laboratory, AFWL-TR-71-50. High Power Gas Laser Technology Forecast, by L.N. Peckham and R.W. Davis, Unclassified, May 1971.

2. Allen, L. and Jones, D.G.C., Principles of Gas Lasers, Plenum Press, 1967.

3. Becker, P.W., Bibliography on Lasers, Becker and Warren, 1962.

4. Bloom, Arnold L., Gas Lasers, J. Wiley, 1968.

5. Boeing Company, Shock Tube Driven Gas Dynamic Laser Development, by J.H. Carr and Others, July 1972.

6. Garrett, C.G.B., Gas Lasers, McGraw-Hill, 1967.

7. General Electric Company Report 9170-015, A Computer Program for the Calculation of Population Inversions and Small Signal Gain For a CO_2, N_2, H_2O, O_2 Gas Dynamics Laser, by D.W. Hall, July 1973.

8. Heard, Harry, G., Laser Parameter Measurements Handbook, Wiley, 1968.

9. Lengyel, B.A., Introduction to Laser Physics, John Wiley and Sons Inc., 1966.

10. Liepmann, H.W. and Roshko, A., Elements of Gas Dynamics, John Wiley and Sons, Inc., 1957.

11. Naval Research Laboratory, NRL High Energy Laser Program Progress Report, 15 January 1972, to 15 April 1972, by Harold W. Gandy and Others, Unclassified, 31 June 1971.

12. RAND Corporation, Report R-779-PR, A New Air Weapon: A Briefing Text, by R.O. Hundly, Unclassified, J-ne 1971.

13. Scripps Institution of Oceanography, SlO Reference 63-5, Airborne Laser Systems, by James L. Harris, Unclassified, February, 1963.

14. Fraas, A.P. and Ozisik, M.N., Heat Exchanger Design, Chapter 4 and 10, Wiley 1965.

15. Sears, F.W., Thermodynamics, p. 248, Addison-Wesley, 1964.

INITIAL DISTRIBUTION

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia, 22314	2
2.	Library, Code 0212 Naval Postgraduate School Monterey, California, 93940	2
3.	Professor Allen E. Fuhs, Chairman Department of Mechanical Engineering Naval Postgraduate School Monterey, California 93940	6
4.	Professor John N. Cooper Department of Physics and Chemistry Naval Postgraduate School Monterey, California 93940	1
5.	Dr. Jim Ortwerth Air Force Weapons Laboratory Kirtland Air Force Base Albuquerque, New Mexico 87117	1
6.	Capt. James Wilson PMS 405 Naval Sea Systems Command Washington, D.C. 20360	1
7.	William Volz Code 320 Naval Air Systems Command Washington, D.C. 20360	1
8.	E.M. Fisher Code 350 Naval Air Systems Command Washington, D.C. 20360	1
9.	Lt. Loran E. Rhine 227 Brettonwood Dr. San Antonio, Texas 78218	4
10.	Lt. Mike Parrish, Section WP-54, SMC 2234 Naval Postgraduate School Monterey, California 93940	1